Direct slope reconstruction algorithm for woofer-tweeter adaptive optics systems

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Abstract: We present a direct slope reconstruction algorithm to control dual-deformable mirror adaptive optics systems. A global response matrix was derived from the response matrices of each deformable mirror. Simulation results validated this control method.

OCIS codes: 000.3860 (Mathematical methods in physics); 010.1080 (Active or adaptive optics)

1. Introduction

Adaptive optics (AO) systems are being used with increased frequency to perform visual psychophysics and high resolution retinal imaging in the living eye. Traditionally, most deformable mirrors used in vision AO systems have small magnitudes of stroke, limiting their use in most eyes to a correction of higher order aberrations and requiring the use of trial lenses to correct the eye's large defocus and astigmatic errors. With continued developments in large stroke deformable mirror technology, vision scientists are now exploring the use of dual deformable mirror systems to fully correct for the eye's lower and higher order aberrations [1-4]. One such approach, implemented in astronomy and vision science [1-8], is the use of a high-stroke (i.e., woofer) deformable mirror to correct for large amplitude, lower order aberrations and a lower-stroke (i.e., tweeter) deformable mirror to compensate for lower amplitude, higher order aberrations.

Several methods have been investigated for controlling dual deformable mirror AO systems, including Zernike modal reconstruction [5], modal decomposition [6, 7], and zonal reconstruction algorithms [4, 8]. The direct slope algorithm is a direct and flexible technique that attempts to correct the wavefront by zeroing the local slope vector across the pupil [9]. To date, it has been challenging to implement this method in dual deformable mirror systems due to coupling between the woofer and tweeter's response matrices. In this paper, we present a direct slope reconstruction algorithm for controlling two deformable mirrors simultaneously. We derive global response and control matrices for a woofer-tweeter system in which the response matrices for the woofer and tweeter are orthogonal to each other.

2. Methods

To efficiently operate a woofer-tweeter system using a direct slope algorithm, it is important to remove any coupling between the woofer and tweeter's response matrices and prevent both deformable mirrors from producing piston, tip and tilt. We assume that the tweeter contains a higher number of actuators (n_t actuators) than the woofer (n_w actuators) over the effective pupil. It is possible to construct a new response matrix for the tweeter, D_t', that is orthogonal to the woofer's response matrix, D_w:

$$D_t' = f(D_t, D_w) \qquad (1)$$

where D_t is the tweeter's original response matrix. To ensure that the tweeter's new response matrix will be orthogonal to the woofer's response matrix, we can first subtract D_{tw}, the projection of the tweeter's original response matrix (D_t) onto the woofer's response matrix (D_w), from the tweeter's original response matrix (D_t):

$$D_t' = D_t - D_{tw} \qquad (2)$$

D_{tw} is given by

$$D_{tw} = D_w P \qquad (3)$$

where P is a matrix containing the coefficients resulting from the projection of D_t onto D_w [10], i.e.

$$P = D_w^{\dagger} D_t \qquad (4)$$

Combining Equations (2)-(4), the tweeter's new response matrix, D_t', can be written as

$$D_t' = D_t - D_w D_w^{\dagger} D_t \qquad (5)$$

This new response matrix is now orthogonal to the woofer's response matrix.

To prevent each deformable mirror from producing piston, we must ensure that the sum of the voltages (V) applied to all actuators is zero. To remove tip and tilt from the correction, the sum of the inner product of each actuator's position vector and the corresponding voltage applied at that actuator must be zero. These constraints are given by

$$\sum_{i=1}^{n_w} V_i = \sum_{i=1}^{n_w} X_i V_i = \sum_{i=1}^{n_w} Y_i V_i = 0 \text{ and } \sum_{i=1}^{n_t} V_i = \sum_{i=1}^{n_t} X_i V_i = \sum_{i=1}^{n_t} Y_i V_i = 0 \quad (6)$$

where X_i and Y_i are the x- and y-positions of each actuator defined in the unit circle. In matrix form, Equation (6) may be written as

$$\begin{bmatrix} 1 & \dots & 1 \\ X_1 & \dots & X_{nw} \\ Y_1 & \dots & Y_{nw} \end{bmatrix} \begin{bmatrix} V_1 \\ \dots \\ V_i \end{bmatrix} = C_w V_w = 0 \text{ and } \begin{bmatrix} 1 & \dots & 1 \\ X_1 & \dots & X_{nt} \\ Y_1 & \dots & Y_{nt} \end{bmatrix} \begin{bmatrix} V_1 \\ \dots \\ V_i \end{bmatrix} = C_t V_t = 0 \qquad (7)$$

where V_t and V_w are the actuator voltages for the tweeter and woofer, respectively. The new global response matrix for the woofer-tweeter system, D, can be written as

$$D = \begin{bmatrix} D_w & D_t' \\ C_w & 0 \\ 0 & C_t \end{bmatrix} \qquad (8)$$

and the new slope vector based on the direct slope algorithm can be expressed as

$$S = DV = \begin{bmatrix} D_w & D_t' \\ C_w & 0 \\ 0 & C_t \end{bmatrix} \begin{bmatrix} V_w \\ V_t \end{bmatrix} = D_w V_w + D_t' V_t \qquad (9)$$

The global control matrix can be obtained by applying Singular-Value-Decomposition to the global response matrix, D, in Equation (8) [11]. Notice that the dimensions for the pseudo-inverse of D should be $(n_w+n_t) \times (n_s+6)$, where n_s is the number of slope measurements across the pupil. The global control matrix can be obtained by extracting a subset of the pseudo-inverse matrix of D of dimensions $(n_w+n_t) \times n_s$. We can calculate the voltages to apply to the woofer and tweeter via

$$[V_w; V_t] = D^{\dagger}{}_{[1:(n_w+n_t),1:n_s]} S \qquad (10)$$

3. Simulation Results

We performed simulations of the direct slope algorithm based on our adaptive optics scanning laser ophthalmoscope (AOSLO) that uses a high-stroke woofer (Mirao52d, Imagine Eyes, Inc, France) and a low-stroke stroke tweeter (Multi-DM, Boston Micromachines, Massachusetts, USA) to correct the eye's aberrations over a 6.6-mm pupil (at the plane of the eye). The woofer has 52 actuators with a pitch of 2.5 mm, a wavefront stroke of 50 μm, and a measured inter-actuator coupling of 0.6. The effective beam diameter on the woofer is 13.2 mm, covering 32 actuators. The tweeter has 140 actuators with a pitch of 450 μm, a wavefront stroke of 2.1 μm, and an inter-actuator coupling of 0.32. The effective pupil diameter on the tweeter is 4.9 mm, covering 120 actuators. 208 subapertures sampled the pupil at the Shack-Hartmann wavefront sensor. The response matrix was measured separately for the woofer and the tweeter.

Random wave aberrations were generated to test the algorithm's correction capability. As an example, one aberration profile consisted of 2.3 Diopters (D) of defocus, -1.9 D of astigmatism, a higher order root-mean-square (RMS) wavefront error of 0.57 μm, and a total RMS wavefront error of 1.58 μm. Based on Equation (9), we obtained the global control matrix using the slope data measured by the Shack-Hartmann. Actuator voltages were calculated using Equation (10) and were simultaneously applied to the woofer and tweeter to compensate for this sample wavefront. The total residual RMS error after the simulated open-loop correction using the direct slope algorithm was 0.09 μm and the Strehl ratio was 0.76.

4. Summary

We presented a direct slope reconstruction algorithm to control a dual-deformable mirror adaptive optics system. A global response matrix was derived from the response matrices of each deformable mirror. Simulation results validated this control method.

5. References

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